

Asteroids, Earth and Elliptical Orbits

Objectives

- Students will demonstrate how to identify and use the correct nomenclature for ellipse parts and identify the equation of an ellipse.
- Students will demonstrate how to manipulate the ellipse equation to solve for variables.
- Students will plot coordinate points obtained from an ellipse and identify major and minor axis.
- Students will relate and infer real-world astronomical pathways and perfect ellipses found in mathematics.
- Students will construct a model of the Earth's orbit and that of the Apophis asteroid and identify where there is potential for collision.

Suggested Grade Level

9th to 12th

Subject areas

Integrated Science
Physical Science
Astronomy
Mathematics Algebra

Timeline

One to two class periods

National Science Content Standards

Science as Inquiry

NS.9-12.1

As a result of activities in grades 9-12, all students should develop

- Abilities necessary to do scientific inquiry
- Understandings about scientific inquiry

Physical Science

NS.9-12.2

As a result of their activities in grades 9-12, all students should develop an understanding of

- Structure of atoms
- Structure and properties of matter
- Chemical reactions
- Motions and forces
- Conservation of energy and increase in disorder
- Interactions of energy and matter

Earth and Space Standards

NS.9-12.4

As a result of their activities in grades 9-12, all students should develop an understanding of

- Energy in the Earth system
- Geochemical cycles
- Origin and evolution of the Earth system
- Origin and evolution of the universe

National Math Content Standards

NM-NUM.9-12.1

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

- develop a deeper understanding of very large and very small numbers and of various representations of them;
- compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions;
- understand vectors and matrices as systems that have some of the properties of the real-number system;
- use number-theory arguments to justify relationships involving whole numbers.

NM-ALG.9-12.2

Represent and analyze mathematical situations and structures using algebraic symbols

- understand the meaning of equivalent forms of expressions, equations, inequalities, and relations;
- write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency -- mentally or with paper and pencil in simple cases and using technology in all cases;
- use symbolic algebra to represent and explain mathematical relationships;
- use a variety of symbolic representations, including recursive and parametric equations, for functions and relations;
- judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

NM-GEO.9-12.2

Specify locations and describe spatial relationships using coordinate geometry and other representational systems

- use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;
- investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

Background

This lesson is appropriate for students studying mathematics or any science topics that have to do with rotational motion, such as astronomy, geology, physical science, biology, technology, and social studies. The students should have prior knowledge of the galactic and solar system models as to how and why objects move in their particular pathway. Students should have an understanding

of variables within math expressions and be able to use their functions in a methodical manner deriving solutions to problems through the methods of substitution, solving for known variables, rewriting equations with the unknown variable isolated to a particular equation, and equation manipulation.

Materials

Pencil, pen, graphing paper, drawing compass, pushpins, cardboard or tack board, thin string

Lesson

1. Have students review the basic math functions and operations; i.e. addition, subtraction, multiplication, division, exponent rules, radicals and taking roots, variables in a problem, converting text to equations, etc.
2. Have students review and/or analyze the orbital paths of the bodies in the solar system and the galaxy. Emphasize Earth's orbit as being nearly a circle (a circle is an ellipse whose major and minor axis are the same).
3. Have students construct circles by tying one end of a piece of string to a push-pin and the other end near the point of their pencil. Place the pin in the center of a piece of cardboard and create a circle by holding the pencil upright 90 degrees to the drawing surface and at the max distance allowable by the string length. Rotate the pencil around the drawing surface creating a circle. Point out to the students that the circle is inscribed line equidistance from a point of origin, the center of the circle. This is the concept of radius. Have students continue drawing circles of varying radii, using a drawing compass to improve the speed and quality of their circles.
4. Have students construct ellipses on cardboard or tack board. An ellipse can be created in a manner similar to the above circle method. Tie a piece of string between two push-pins. Place the push-pins into the cardboard at a distance less than the total length of the string between them. This will create a slack or loose string upon the drawing surface. With a pen or pencil held upright and at 90 degrees to the drawing surface engage the loose string and pull it taut by having the pencil tip rest against and be limited by the string, and drawing surface. Draw a line on the cardboard at the limit of the strings loop moving around and to the outside of the pins. Unlike the circle, you will have to ensure that the string does not become entangled around the pins. Have students repeat the above process varying the distance of the two pins. Notice as the two pins get closer together the ellipse becomes more circular. An ellipse has two focal points or centers while a circle has only one. A line drawn through these two centers will create what is termed the major axis. A line drawn perpendicular from the center of this line is called the minor axis. Using

the equation of an ellipse we can calculate the points of inflection at these four points and draw any ellipse.

5. The equation for a circle with its center at the origin (coordinates (0,0)) is as follows: $X^2 + Y^2 = r^2$, where "r" is the radius and "X" and "Y" are axis drawn at 90 degrees to each other through the origin. The standard form of the equation for an ellipse whose center is the origin is $X^2/R^2 + Y^2/R^2 = 1$. Since there are two different foci, we denote these with "A" and "B", respectfully, yielding the equation: $X^2/A^2 + Y^2/B^2 = 1$. Taking the square root of "A" and "B" produces the distance from the origin to the inflection points along the ellipse at the intersection of the major and minor axis, and we can then plot these points to determine the limit of the ellipse. These points will be (A,0) (-A,0) (B,0) (-B,0). For example, in the equation of an ellipse;

$$X^2 / 25 + Y^2 / 16 = 1$$

We take the square root of 25 and 16 to get (+ or -) 5 and 4 giving us the points (5,0) (-5,0) (4,0) (-4,0). Plotting these points out on a coordinate system allows us to define where the ellipse exists.

6. Have the students solve for the following points and graph the ellipse using the created coordinates;

$$X = [25, 9, 64, 4]$$

$$Y = [4, 16, 36, 9]$$

7. Construct a graph where the Earth has a circular orbit around the sun with a radius of 25 and Apophis has an elliptical orbit around the sun with radiuses of its foci at 36 and 16. Find the length of the major and minor axis, graph the limits of the ellipse, and interpret at what coordinates the asteroids path will cross the Earth's path, if any, if they existed within the same celestial plane.

Extensions

Students can expound and enrich their experience in a number of ways. Construct a three dimensional model of the solar system including the Apophis asteroid. Write a journal entry detailing observations and expectations gleaned from the above activity. Conduct a Socratic Seminar on the feasibility of deflecting space-based objects from interacting with the Earth system; consider nuclear strikes, landed platform motors, tethers, etc. Using GIS imagery, satellite photos, and Web-based data on impact sites, interpret where past asteroids and meteorites have struck the Earth. Contemplate and discuss why there is a low amount of occurrences with that of craters compared to other solar bodies i.e. the moon, mars, and moons of the jovial planets.

Evaluation

- Students successfully identify and can define the terms asteroid, meteorite, foci, origin, axis, radius, ellipse, and orbit.
- Students construct reasonable facsimiles of circles and ellipses using the instructed techniques.
- Students accurately solve for various values in determination of an ellipses' major and minor axis, graph these coordinate points and construct ellipses.
- Students construct reasonable connections between mathematical elliptical orbits and the actual orbits of the Earth and Apophis.
- Students successfully construct a model of the Earth and Apophis orbits including potential striking points.

Resources

<http://www.spaceref.com/news/viewsr.html?pid=17666>

http://observe.arc.nasa.gov/nasa/education/teach_guide/craters.html

http://observe.arc.nasa.gov/nasa/exhibits/craters/impact_home.html

<http://deepimpact.jpl.nasa.gov/educ/DesigningCraters.html>

<http://www.astrosociety.org/education/activities/astroacts08.html>